Annals of **F**uzzy **M**athematics and **I**nformatics
Volume x, No. x, (Month 201y), pp. 1–xx
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

Fuzzy soft α-ψ-contractive type mappings and some fixed point theorems in fuzzy soft metric spaces

A. F. Sayed, A. Alahmari

Received 26 November 2017; Accepted 13 December 2017

Abstract. In this paper, we introduce the notions of fuzzy soft $\alpha-\psi$-contractive mappings and cyclic fuzzy soft $(\alpha, \beta)-\psi$-contractive mappings. Finally, we prove some fixed point theorems in fuzzy soft metric spaces.

2010 AMS Classification: 03E72, 47H10, 54A40, 54E50, 54E40

Keywords: Soft set, Fuzzy set, Fuzzy soft set, Fuzzy soft metric space, fuzzy soft contractive mapping.

Corresponding Author: A. F. Sayed (dr.afsayed@hotmail.com)

1. Introduction

In daily life, the problems in many fields deal with uncertain data and are not successfully modelled in classical mathematics. There are two types of mathematical tools to deal with uncertainties namely fuzzy set theory introduced by Zadeh [41] and the theory of soft sets initiated by Molodstov [28] which helps to solve problems in all areas. After than , the properties and applications on this theorem have been studied by many authors (e.g. [3], [7], [8], [14], [22], [36], [42]). Maji et al. [22] introduced several operations in soft sets and has also coined fuzzy soft sets. Many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets (e.g. [2], [4], [9], [10], [13], [23], [24], [32], [37], [38], [40]). In [5] Beaula et al. were introduced a definition of the fuzzy soft metric space, also in [35] we introduced a concept of fuzzy Soft contractive mappings in fuzzy soft metric spaces and studied some fixed point theorems.
The Banach contraction principle is certainly a classical result of modern analysis. This principle has been extended and generalized in different directions in metric spaces [39]. In (1988), Grabiec [17] initiated the study of the fixed point theory in fuzzy metric space. In (2002), Gregori and Sapena [18] introduced new kind of contractive mappings in modified fuzzy metric spaces and proved a fuzzy version of Banach contraction principle (see [25], [26], [31]). In particular, Mihet [27] introduced the concepts of fuzzy ψ-contractive mappings which enlarge the class of fuzzy

contractions in Gregori and Sapena [18] and many authors Abbas et al. [1] and Hong [19] have used the result of Mihet [27]. In (2012) Samet et al. [34] introduced the concept of $\alpha-\psi$-contractive mapping and utilized the same concept to prove several interesting fixed point theorems in setting of metric spaces (see [2], [15], [16], [29], [30], [39]). In this paper, we introduce a concept of fuzzy soft $\alpha-\psi$-contractive type mappings and cyclic fuzzy soft $(\alpha,\beta)-\psi$-contractive mappings, and establish fixed point theorems for such mappings in complete fuzzy soft metric spaces. Starting from the Banach contraction principle, the presented theorems are the extension, generalization, and improvement of many existing results in the literature.

2. Preliminaries

In this section we present some basic definitions of fuzzy soft set and fuzzy soft metric space.
Throughout our discussion, X refers to an initial universe, E the set of all parameters for X, $P(X)$ denotes the power set of X and $I=[0,1]$.

Definition 2.1 ([41]). A fuzzy set A in a non-empty set X is characterized by a membership function $\mu_A : X \to I$ whose value $\mu_A(x)$ represents the "degree of membership" of x in A for $x \in X$. Let I^X denotes the family of all fuzzy sets on X.

A member A in I^X is contained in a member B of I^X, denoted by $A \leq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ for every $x \in X$ (see [41]).

Let $A, B \in I^X$, we have the following fuzzy sets (see [41]).

(i) Equality: $A = B$ if and only if $\mu_A(x) = \mu_B(x)$, for all $x \in X$.
(i) Intersection: $C = A \wedge B \in I^X$ by $\mu_C(x) = min\{\mu_A(x), \mu_B(x)\}$, for all $x \in X$.
(iii) Union: $D = A \vee B \in I^X$ by $\mu_C(x) = max\{\mu_A(x), \mu_B(x)\}$, for all $x \in X$.
(iv) Complement: $E = A^c \in I^X$ by $\mu_E(x) = 1 - \mu_A(x)$, for all $x \in X$.

Definition 2.2 ([41]). The empty fuzzy set, denoted by $\tilde{0}$, is the function which maps each $x \in X$ to 0. That is, $\tilde{0}(x) = 0$, for all $x \in X$. A universal fuzzy set denoted by $\tilde{1}$ is a function which maps each $x \in X$ to 1. That is, $\tilde{1}(x) = 1$, for all $x \in X$.

Definition 2.3 ([28]). Let $A \subseteq E$. A pair (F, A) is called a soft set over (X, E), if F is a mapping $F : A \to P(X)$.

In other words, a soft set over (X, E) is a parameterized family of subsets of the universe X. For $\epsilon \in A$, $A(\epsilon)$ may be considered as the set of ϵ-approximate elements of the soft set (F, A), or as the set of ϵ-approximate elements of the soft set.

Definition 2.4 ([21]). Let $A \subseteq E$. Then a pair (f, A), denoted by f_A, is called a fuzzy soft set over (X, E), where f is a mapping given by $f : A \to I^X$ defined by $f_A(e) = \mu^e_{f_A}$, where

$$\mu^e_{f_A} = \begin{cases} \tilde{0}, & \text{if } e \notin A, \\ otherwise, & \text{if } e \in A. \end{cases}$$

$\widetilde{(X,E)}$ denotes the class of all fuzzy soft sets over (X,E) and is called a fuzzy soft universe ([22]).

Definition 2.5 ([23])**.** A fuzzy soft set f_A over (X,E) is said to be:

(a) NULL fuzzy soft set, denoted by $\tilde{\phi}$, if for all $e \in A, f_A(e) = \tilde{0}$,
(b) absolute fuzzy soft set, denoted by $\tilde{E}$, if for all $e \in A, f_A(e) = \tilde{1}$.

Definition 2.6 ([33])**.** The complement of a fuzzy soft set (f,A), denoted by $(f,A)^c$, is defined by $(f,A)^c = (f^c, A), f_A^c : E \to I^X$ is a mapping given by $\mu^e_{f_A^c} = \tilde{1} - \mu^e_{f_A}$ where $\tilde{1}(x) = 1$, for all $x \in X$. Clearly, $(f_A^c)^c = f_A$.

Definition 2.7 ([33])**.** Let $f_A, g_B \in \widetilde{(X,E)}$. Then f_A is called a fuzzy soft subset of g_B, denoted by $f_A \tilde{\subseteq} g_B$, if $A \subseteq B$ and $\mu^e_{f_A} \leq \mu^e_{g_B}$, for all $e \in A$, i.e. $\mu^e_{f_A}(x) \leq \mu^e_{g_B}(x)$, for all $x \in X$ and for all $e \in A$.

Definition 2.8 ([33])**.** Let $f_A, g_B \in \widetilde{(X,E)}$. Then the union of f_A and g_B is also a fuzzy soft set h_C, where $C = A \cup B$ and for all $e \in C, h_C(e) = \mu^e_{h_c} = \mu^e_{f_A} \vee \mu^e_{g_B}$. Here we write $h_C = f_A \tilde{\cup} g_B$.

Definition 2.9 ([33])**.** Let $f_A, g_B \in \widetilde{(X,E)}$. Then the intersection of f_A and g_B is also a fuzzy soft set d_C, where $C = A \cap B$ and for all $e \in C, d_C(e) = \mu^e_{d_c} = \mu^e_{f_A} \wedge \mu^e_{g_B}$. Here we write $d_C = f_A \tilde{\cap} g_B$.

Definition 2.10 ([20])**.** The fuzzy soft set $f_A \in \widetilde{(X,E)}$ is called fuzzy soft point, if there exist $x \in X$ and $e \in E$ such that $\mu^e_{f_A}(x) = \alpha (0 < \alpha \leq 1)$ and $\mu^e_{f_A}(y) = 0$ for each $y \in X - \{x\}$, and this fuzzy soft point is denoted by x^e_α or f_e.

Definition 2.11 ([20])**.** The fuzzy soft point x^e_α is said to be belonging to the fuzzy soft set (g,A), denoted by $x^e_\alpha \tilde{\in} (g,A)$, if for the element $e \in A$,$\alpha \leq \mu^e_{g_A}(x)$.

Definition 2.12 ([5])**.** Let f_A be fuzzy soft set over (X,E). Then two fuzzy soft points $F_{e_1}, F_{e_2} \in f_A$ are said to be equal, if $\mu_{f_{e_1}}(x) = \mu_{f_{e_2}}(x)$, for all $x \in X$. Thus $f_{e_1} \neq f_{e_2}$ if and only $\mu_{f_{e_1}}(x) \neq \mu_{f_{e_2}}(x)$, for all $x \in X$.

Proposition 2.13 ([5])**.** *The union of any collection of fuzzy soft points can be considered as a fuzzy soft set and every fuzzy soft set can be expressed as the union of all fuzzy soft points as*

$$f_A = \{\tilde{\cup}_{f_e \tilde{\in} f_A} f_e : e \in E\}.$$

Proposition 2.14 ([5])**.** *Let f_A, f_B be two fuzzy soft sets then $f_A \tilde{\subseteq} f_B$ if and only if $f_e \tilde{\in} f_A$ implies $f_e \tilde{\in} f_B$ and hence $f_A = f_B$ if and only if $f_e \tilde{\in} f_A$ and only if $f_e \tilde{\in} f_B$.*

Definition 2.15 ([12])**.** Let $\mathbb{R}$ be the set of real numbers and $B(\mathbb{R})$ be the collection of all non-empty bounded subsets of $\mathbb{R}$ and E be taken as a set of parameters, $A \subseteq E$. Then a mapping $f : A \to B(\mathbb{R})$ is called a soft real set. If a soft real set is a singleton soft set, it will be called a soft real number and denoted by $\tilde{r}, \tilde{s}, \tilde{t}$ etc. $\tilde{0}$ and $\tilde{1}$ are the soft real numbers, where $\tilde{0}(e) = 0, \tilde{1}(e) = 1$, for all $e \in E$, respectively.

The set of all soft real numbers is denoted by $\mathcal{R}(A)$ and the set of all non-negative soft real numbers by $\mathcal{R}(A)^*$

Definition 2.16 ([11]). A (non negative) fuzzy soft real number is a fuzzy set on the set of all (non negative) soft real numbers $\mathcal{R}(A)$, that is, a mapping $\tilde{\tilde{\lambda}} : \mathcal{R}(A) \to [0,1]$, associating with each (non negative) soft real number $\tilde{t}$, its grade of membership $\tilde{\tilde{\lambda}}(\tilde{t})$ satisfying the following conditions:

(i) $\tilde{\tilde{\lambda}}$ is convex,
that is, $\tilde{\tilde{\lambda}}(\tilde{t}) \geq \min\ (\tilde{\tilde{\lambda}}(\tilde{s}), \tilde{\tilde{\lambda}}(\tilde{r}))$ for $\tilde{s} \subseteq \tilde{t} \subseteq \tilde{r}$,

(ii) $\tilde{\tilde{\lambda}}$ is normal,
that is, there exists $\tilde{t_0} \in \mathcal{R}(A)^*$ such that $\tilde{\tilde{\lambda}}(\tilde{t_0}) = 1$,

(iii) $\tilde{\tilde{\lambda}}$ is upper semi continuous provided for all $\tilde{t} \in \mathcal{R}(A)$ and $\alpha \in [0,1]$, $\tilde{\tilde{\lambda}}(\tilde{t}) < \alpha$, there is a $\delta > 0$ such that $\parallel \tilde{s} - \tilde{t} \parallel \leq \delta$ implies that $\tilde{\tilde{\lambda}}(\tilde{s}) < \alpha$.

Let $A \subseteq E$ and the collection of all fuzzy soft points of a fuzzy soft set f_A over (X,E) be denoted by $FSC(f_A)$.

Let $\mathcal{R}(A)^*$ be the set of all non negative fuzzy soft real numbers. The fuzzy soft metric using fuzzy soft points is defined as follows:

Definition 2.17 ([5]). Let $A \subseteq E$ and $\tilde{E}$ be the absolute fuzzy soft set. A mapping $\tilde{d} : FSC(\tilde{E}) \times FSC(\tilde{E}) \to \mathcal{R}(A)^*$ is said to be a fuzzy soft metric on $\tilde{E}$, if $\tilde{d}$ satisfies the following conditions:

$(FSM_1) : \tilde{d}(f_{e_1}, f_{e_2}) \tilde{\geq} \tilde{0}$, for all $f_{e_1}, f_{e_2} \tilde{\in} \tilde{E}$,
$(FSM_2) : \tilde{d}(f_{e_1}, f_{e_2}) = \tilde{0}$ if and only if $f_{e_1} = f_{e_2}$, for al $f_{e_1}, f_{e_2} \tilde{\in} \tilde{E}$,
$(FSM_3) : \tilde{d}(f_{e_1}, f_{e_2}) = \tilde{d}(f_{e_2}, f_{e_1})$, for all $f_{e_1}, f_{e_2} \tilde{\in} \tilde{E}$,
$(FSM_4) : \tilde{d}(f_{e_1}, f_{e_3}) \tilde{\leq}, \tilde{d}(f_{e_1}, f_{e_2}) + \tilde{d}(f_{e_2}, f_{e_3})$ for all $f_{e_1}, f_{e_2}, f_{e_3} \tilde{\in} \tilde{E}$).

The fuzzy soft set $\tilde{E}$ with the fuzzy soft metric $\tilde{d}$ is called the fuzzy soft metric space and is denoted by $(\tilde{E}, \tilde{d})$.

Definition 2.18 ([6]). Let $(\tilde{E}, \tilde{d})$ be a fuzzy soft metric space and $\tilde{\tilde{t}}$ be a fuzzy soft real number and $\tilde{\epsilon} \in (0,1)$. A fuzzy soft open ball centered at the fuzzy point $f_e \tilde{\in} \tilde{E}$ and radius $\tilde{\tilde{t}}$ is a collection of all fuzzy soft points g_e of $\tilde{E}$ such that $\tilde{d}(g_e, f_e) \tilde{<} \tilde{\tilde{t}}$. It is denoted by $\tilde{\tilde{B}}(f_e, \tilde{\tilde{t}}, \tilde{\epsilon})$, where $\tilde{\tilde{B}}(f_e, \tilde{\tilde{t}}, \tilde{\epsilon}) = \{g_e \tilde{\in} \tilde{E} | \tilde{d}(g_e, f_e) \tilde{<} \tilde{\tilde{t}}\}$ with $\mid \mu^a_{g_e}(x) - \mu^a_{f_e}(x) \mid < \tilde{\epsilon}$, for all $a \in E, x \in X$.

The fuzzy soft closed ball is denoted by $\tilde{\tilde{B}}[f_e, \tilde{\tilde{t}}, \tilde{\epsilon}] = \{g_e \tilde{\in} \tilde{E} | \tilde{d}(g_e, f_e) \tilde{\leq} \tilde{\tilde{t}}\}$ with $\mid \mu^a_{g_e}(x) - \mu^a_{f_e}(x) \mid \leq \tilde{\epsilon}$, for all $a \in E, x \in X$.

Definition 2.19 ([6]). A sequence $\{f_{e_n}\}$ in a fuzzy soft metric space $(\tilde{E}, \tilde{d})$ is said to converge to $f_{e'}$, if $\tilde{d}(f_{e_n}, f_{e'}) \to \tilde{0}$ as $n \to \infty$, for every $\tilde{\epsilon} \tilde{>} \tilde{0}$, there exists $\tilde{\delta} \tilde{>} \tilde{0}$ and a positive integer $N = N(\tilde{\epsilon})$ such that $\tilde{d}(f_{e_n}, f_{e'}) \tilde{<} \tilde{\delta}$ implies $\mid \mu^a_{f_{e_n}}(x) - \mu^a_{f_{e'}}(x) \mid < \tilde{\epsilon}$, whenever $n \geq N, a \in E$ and $x \in X$. It is usually denoted as $\lim_{\ n \to \infty} f_{e_n} = f_{e'}$.

Definition 2.20 ([6]). A sequence $\{f_{e_n}\}$ in a fuzzy soft metric space $(\tilde{E}, \tilde{d})$ is said to be a Cauchy sequence, if to every $\tilde{\epsilon} \tilde{>} \tilde{0}$, there exists $\tilde{\delta} \tilde{>} \tilde{0}$ and a positive integer $N = N(\tilde{\epsilon})$ such that $\tilde{d}(f_{e_n}, f_{e_m}) \tilde{<} \tilde{\delta}$ implies $\mid \mu^a_{f_{e_n}}(x) - \mu^a_{f_{e_m}}(x) \mid < \tilde{\epsilon}$, for all $n, m \geq N, a \in E$ and $x \in X$, that is, $\tilde{d}(f_{e_n}, f_{e_m}) \to \tilde{0}$ as $n, m \to \infty$.

Definition 2.21 ([6]). A fuzzy soft metric space $(\tilde{E},\tilde{d})$ is said to be complete, if every cauchy sequence in $\tilde{E}$ converges to some fuzzy soft point of $\tilde{E}$.

Definition 2.22 ([35]). Let $(\tilde{E},\tilde{d})$ and $(\tilde{E}',\tilde{\rho})$ be two fuzzy soft metric spaces. Then the mapping $\varphi_\psi = (\varphi,\psi) : (\tilde{E},\tilde{d}) \to (\tilde{E}',\tilde{\rho})$ is called a fuzzy soft mapping, if $\varphi : \tilde{E} \to \tilde{E}'$ and $\psi : E \to E'$ are two mappings.

Definition 2.23 ([35]). Let $(\tilde{E},\tilde{d})$ and $(\tilde{E}',\tilde{\rho})$ be two fuzzy soft metric spaces. Then $\varphi_\psi : (\tilde{E},\tilde{d}) \to (\tilde{E}',\tilde{\rho})$ is called a fuzzy soft continuous mapping at the fuzzy soft point $f_e \tilde{\in} FSC(\tilde{E})$, if for every fuzzy soft open ball $\tilde{\tilde{B}}(\varphi_\psi(f_e),\tilde{\tilde{t}},\tilde{\epsilon})$ of $(\tilde{E}',\tilde{\rho})$, there exists a fuzzy soft open ball $\tilde{\tilde{B}}(f_e,\tilde{\tilde{t}},\tilde{\epsilon})$ of $(\tilde{d},\tilde{E})$ such that $\varphi(\tilde{\tilde{B}}(f_e,\tilde{\tilde{t}},\tilde{\epsilon})) \tilde{\subseteq} \tilde{\tilde{B}}(\varphi_\psi(f_e),\tilde{\tilde{r}},\tilde{\epsilon})$.

If $\varphi_\psi(f_e)$ is a fuzzy soft continuous mapping at every fuzzy soft point f_e of $(\tilde{E},\tilde{d})$, then it is said to be fuzzy soft continuous mapping on $(\tilde{E},\tilde{d})$.

Now, this definition can be expressed using $\tilde{\varepsilon} - \tilde{\delta}$ as follows:

Definition 2.24 ([35]). The mapping $\varphi_\psi : (\tilde{E},\tilde{d}) \to (\tilde{E}',\tilde{\rho})$ is said to be a fuzzy soft continuous mapping at the fuzzy soft point $f_e \tilde{\in} FSC(\tilde{E})$, if for every $\tilde{\varepsilon} \tilde{>} \tilde{0}$, there exists a $\tilde{\delta} \tilde{>} \tilde{0}$ such that $\tilde{d}(f_e, g_{\acute{e}}) \tilde{<} \tilde{\delta}$ implies that $\tilde{\rho}(\varphi_\psi(f_e), \varphi_\psi(g_{\acute{e}})) \tilde{<} \tilde{\varepsilon}$.

Definition 2.25 ([35]). The fuzzy soft mapping $\varphi_\psi : (\tilde{E},\tilde{d}) \to (\tilde{E}',\tilde{\rho})$ is said to be fuzzy soft sequentially continuous at the fuzzy soft point $f_e \tilde{\in} FSC(\tilde{E})$, if for every sequence of fuzzy soft points $\{f_{e_n}\}$ converging to the fuzzy soft point f_e in the fuzzy soft metric space $(\tilde{E},\tilde{d})$, the sequence $\varphi_\psi(\{f_{e_n}\})$ in $(\tilde{E}',\tilde{\rho})$ converges to a fuzzy soft point $\varphi_\psi(f_e) \tilde{\in} FSC(\tilde{E}')$.

Theorem 2.26 ([35]). *Fuzzy soft continuity is equivalent to fuzzy soft sequential continuity in fuzzy soft metric spaces.*

3. Fuzzy soft $\alpha - \psi$-contractive type mappings and some fixed point theorems

Definition 3.1. Let $T : FSC(\tilde{E}) \to FSC(\tilde{E})$ and $\alpha : FSC(\tilde{E}) \times FSC(\tilde{E}) \to \mathcal{R}(E)^*$. Then we say that T is a fuzzy soft α-admissible, if $f_e, g_{\acute{e}} \tilde{\in} FSC(\tilde{E})$,

$$\alpha(f_e, g_{\acute{e}}) \tilde{\leq} \tilde{1} \Rightarrow \alpha(Tf_e, Tg_{\acute{e}}) \tilde{\leq} \tilde{1}.$$

Example 3.2. Let $X = E = [0,\infty)$ and $T : FSC(\tilde{E}) \to FSC(\tilde{E})$ is defined by $T(f_e) = f_e$, for all $f_e \tilde{\in} FSC(\tilde{E})$. Let $\alpha : FSC(\tilde{E}) \times FSC(\tilde{E}) \to \mathcal{R}(E)^*$ be defined by:

$$\alpha(f_e, g_{\acute{e}}) = \begin{cases} 0.2, & \text{if } f_e \neq g_{\acute{e}}, \\ \tilde{0}, & \text{if } f_e = g_{\acute{e}}. \end{cases}$$

Then T is a fuzzy soft α-admissible.

Definition 3.3. Let Ψ be the family of functions $\psi : \mathcal{R}(E)^* \to \mathcal{R}(E)^*$ satisfying the following conditions:

(i) ψ is non-decreasing,
(ii) $\sum_{n=1}^{+\infty} \psi^n(\tilde{\tilde{t}}) = \tilde{1}$, for all $\tilde{\tilde{t}} \tilde{>} \tilde{0}$, where ψ^n is the n^{th} iterative of ψ.

Remark 3.4. For every function $\psi : \mathcal{R}(E)^* \to \mathcal{R}(E)^*$, the following holds:
if ψ is non-decreasing, then for each $\tilde{\tilde{t}} \tilde{>} \tilde{0}$,

$$lim_{n\to\infty}\psi^n(\tilde{\tilde{t}}) = \tilde{0} \Rightarrow \psi(\tilde{\tilde{t}}) \tilde{<} \tilde{\tilde{t}} \Rightarrow \psi(\tilde{0}) = \tilde{0}.$$

Therefore if $\psi \in \Psi$ then for each $\tilde{\tilde{t}} \tilde{>} \tilde{0}, \psi(\tilde{\tilde{t}}) \tilde{<} \tilde{\tilde{t}} \Rightarrow \psi(\tilde{0}) = \tilde{0}$.

Definition 3.5. Let $(\tilde{E}, \tilde{d})$ be a fuzzy soft metric space and $(T, \varphi) : (\tilde{E}, \tilde{d}) \to (\tilde{E}, \tilde{d})$ be a given fuzzy soft mapping. Then we say that (T, φ) is fuzzy soft $\alpha-\psi$-contractive mapping, if there exists two fuzzy soft functions $(\alpha, \phi) : FSC(\tilde{E}) \times FSC(\tilde{E}) \to \mathcal{R}(E)^*$ and $\psi \in \Psi$ such that

$$(\alpha, \phi)(f_e, g_{\acute{e}})\tilde{d}((T, \varphi)f_e, (T, \varphi)g_{\acute{e}}) \tilde{\leq} \psi(\tilde{d}(f_e, g_{\acute{e}})), \forall f_e, g_{\acute{e}} \in FSC(\tilde{E}). \tag{3.1}$$

Theorem 3.6. *Let $(\tilde{E}, \tilde{d})$ be a complete fuzzy soft metric space. Let (T, φ) be a fuzzy soft $\alpha - \psi$-contractive mapping from $(\tilde{E}, \tilde{d})$ into itself satisfying the following:*
(i) *(T, φ) is a fuzzy soft α-admissible,*
(ii) *there exists $f^0_{e_0} \tilde{\in} FSC(\tilde{E})$ such that $(\alpha, \phi)(f^0_{e_0}, (T, \varphi)f^0_{e_0}) \tilde{\leq} \tilde{1}$,*
(iii) *(T, φ) is fuzzy soft continuous.*

Then (T, φ) has a fixed point, that is, there exists $f_e \in FSC(\tilde{E})$ such that

$$(T, \varphi)f_e = f_e.$$

Proof. Let $f^0_{e_0} \tilde{\in} FSC(\tilde{E})$ such that $(\alpha, \phi)(f^0_{e_0}, (T, \varphi)f^0_{e_0}) \tilde{\leq} \tilde{1}$. Define the sequence $\{f^n_{e_n}\} in (\tilde{E}, \tilde{d})$ by:

$$f^{n+1}_{e_{n+1}} = (T, \varphi)f^n_{e_n}, \forall n \in \mathbb{N}.$$

If $f^n_{e_n} = f^{n+1}_{e_{n+1}}$, for some $n \in \mathbb{N}$, then $f_e = f^n_{e_n}$ is a fixed point of (T, φ).

Assume that $f^n_{e_n} \neq f^{n+1}_{e_{n+1}}$, for all $n \in \mathbb{N}$. Since T is a fuzzy soft α-admissible, we have $(\alpha, \phi)(f^0_{e_0}, f^1_{e_1}) = (\alpha, \phi)(f^0_{e_0}, (T, \varphi)f^0_{e_0}) \tilde{\leq} \tilde{1}$. By induction, we get

$$(\alpha, \phi)(f^0_{e_0}, f^{n+1}_{e_{n+1}}) \tilde{\leq} \tilde{1}, \forall n \in \mathbb{N}. \tag{3.2}$$

Applying the inequality (3.1) with $f_e = f^{n+1}_{e_{n+1}}$ and $g_{\acute{e}} = f^n_{e_n}$, and using (3.2), we obtain

$$\begin{aligned}
\tilde{d}(f^n_{e_n}, f^{n+1}_{e_{n+1}}) &= \tilde{d}((T, \varphi)f^{n-1}_{e_{n-1}}, (T, \varphi)f^n_{e_n}) \\
&\tilde{\leq} (\alpha, \phi)(f^{n-1}_{e_{n-1}}, f^n_{e_n})\tilde{d}((T, \varphi)f^{n-1}_{e_{n-1}}, (T, \varphi)f^n_{e_n}) \\
&\tilde{\leq} \psi(\tilde{d}(f^{n-1}_{e_{n-1}}, f^n_{e_n})).
\end{aligned}$$

By induction, we get

$$\tilde{d}(f^n_{e_n}, f^{n+1}_{e_{n+1}}) \tilde{\leq} \psi^n(\tilde{d}(f^0_{e_0}, f^1_{e_1})), \forall n \in \mathbb{N}. \tag{3.3}$$

From the inequality (3.3) and using the triangular inequality and for $n, m \in \mathbb{N}$ with $m > n$,

$$\tilde{d}(f^n_{e_n}, f^m_{e_m}) \tilde{\leq} \tilde{d}(f^n_{e_n}, f^{n+1}_{e_{n+1}}) + \tilde{d}(f^{n+1}_{e_{n+1}}, f^{n+2}_{e_{n+2}}) + ... + \tilde{d}(f^{m-1}_{e_{m-1}}, f^m_{e_m})$$

$$= \sum_{k=n}^{m-1} \tilde{d}(f_{e_k}^k, f_{e_{k+1}}^{k+1})$$

$$\tilde{\leq} \sum_{k=n}^{m-1} \psi^k(\tilde{d}(f_{e_0}^0, f_{e_1}^1))$$

$$\tilde{\leq} \sum_{k=n}^{+\infty} \psi^k(\tilde{d}(f_{e_0}^0, f_{e_1}^1)).$$

Letting $k \to \infty$, we obtain $\{f_{e_n}^n\}$ is a Cauchy sequence in fuzzy soft metric space in $(\tilde{E}, \tilde{d})$. Since $(\tilde{E}, \tilde{d})$. is complete, there exists $f_e \tilde{\in} FSC(\tilde{E})$ such $f_{e_n}^n \to f_e$ as $n \to \infty$. From the fuzzy soft continuity of (T, φ), it follows that $f_{e_{n+1}}^{n+1} = (T, \varphi) f_{e_n}^n \to (T, \varphi) f_e$ as $n \to \infty$. By the uniqueness of the limit, we get $f_e = (T, \varphi) f_e$. □

Theorem 3.7. *Let $(\tilde{E}, \tilde{d})$ be a fuzzy soft metric space . Let (T, φ) be a fuzzy soft $\alpha - \psi$-contractive mapping from $(\tilde{E}, \tilde{d})$ into itself satisfying the following:*

(i) *(T, φ) is a fuzzy soft α-admissible,*
(ii) *there exists $f_{e_0}^0 \tilde{\in} FSC(\tilde{E})$ such that $(\alpha, \phi)(f_{e_0}^0, (T, \varphi) f_{e_0}^0) \tilde{\leq} \tilde{1}$,*
(iii) *If $\{f_{e_n}^n\}$ is a sequence in $(\tilde{E}, \tilde{d})$ such that $(\alpha, \phi)(f_{e_n}^n, f_{e_{n+1}}^{n+1}) \tilde{\leq} \tilde{1}$, for all $n \in \mathbb{N}$ and $f_{e_n}^n \to f_e \tilde{\in} FSC(\tilde{E})$ as $n \to +\infty$, then $(\alpha, \phi)(f_{e_n}^n, f_e) \tilde{\leq} \tilde{1}$, for all $n \in \mathbb{N}$.*

Then (T, φ) has a fixed point.

Proof. Following the proof of Theorem 3.6, we know that $\{f_{e_n}^n\}$ is a Cauchy sequence in complete fuzzy soft metric space $(\tilde{E}, \tilde{d})$. Then, there exists $f_e \tilde{\in} FSC(\tilde{E})$ such that $f_{e_n}^n \to f_e$ as as $n \to +\infty$.

On the other hand, from inequality (3.2) and the hypothesis (iii), we have

$$(\alpha, \phi)(f_{e_n}^n, f_e) \tilde{\leq} \tilde{1}, \forall n \in \mathbb{N}. \tag{3.4}$$

Now, using the triangular inequality, and the two inequalities (3.1) and (3.4),

$$\tilde{d}((T, \varphi) f_e, f_e) \tilde{\leq} \tilde{d}((T, \varphi) f_{e_n}^n, (T, \varphi) f_e) + \tilde{d}(f_{e_{n+1}}^{n+1}, f_e)$$

$$\tilde{\leq} (\alpha, \varphi)(f_{e_n}^n, f_e) \tilde{d}((T, \varphi) f_{e_n}^n, (T, \varphi) f_e) + \tilde{d}(f_{e_{n+1}}^{n+1}, f_e)$$

$$\tilde{\leq} \psi(\tilde{d}(f_{e_n}^n, f_e)) + \tilde{d}(f_{e_{n+1}}^{n+1}, f_e).$$

Letting $n \to \infty$, since ψ is continuous at $\tilde{0}$, we obtain $\tilde{d}((T, \varphi) f_e, f_e) = \tilde{0}$, that is $(T, \varphi) f_e = f_e$. □

To assure the uniqueness of the fixed point, we will consider the following hypothesis.

(S): For all $f_e, g_{\acute{e}} \tilde{\in} FSC(\tilde{E})$, there exists $h_{e^*} \tilde{\in} FSC(\tilde{E})$ such that $(\alpha, \phi)(f_e, h_{e^*}) \tilde{\leq} \tilde{1}$ and $(\alpha, \phi)(g_{\acute{e}}, h_{e^*}) \tilde{\leq} \tilde{1}$.

Theorem 3.8. *Adding condition (S) to the hypotheses of Theorem 3.6 (resp. Theorem 3.7), we obtain uniqueness of the fixed point of (T, φ).*

Proof. Suppose f_e and $g_{\acute{e}}$ are two fixed points of (T, φ). From (S), there exists $h_{e^*} \tilde{\in} FSC(\tilde{E})$ such that

$$(\alpha, \phi)(f_e, h_{e^*}) \tilde{\leq} \tilde{1}, \qquad (\alpha, \phi)(g_{\acute{e}}, h_{e^*}) \tilde{\leq} \tilde{1}. \tag{3.5}$$

Since (T,φ) is fuzzy soft α-admissible, from inequality 3.5, we get

$$(3.6)\qquad (\alpha,\phi)(f_e,(T,\varphi)^n h_{e^*})\tilde{\leq}\tilde{1}, \qquad (\alpha,\phi)(g_{\acute{e}},(T,\varphi)^n h_{e^*})\tilde{\leq}\tilde{1}, \forall n\in\mathbb{N}.$$

Define the sequence $\{h^n_{e^*_n}\}$ in $(\tilde{E},\tilde{d})$ by $h^{n+1}_{e^*_{n+1}} = (T,\varphi)h^n_{e^*_n}$,for all $n>0$ and $h^0_{e^*_0} = h_{e^*}$.

Using the two inequalities (3.6) and (3.1), we have

$$\begin{aligned}\tilde{d}(f_e,h^{n+1}_{e^*_{n+1}}) &= \tilde{d}((T,\varphi)f_e,(T,\varphi)h^{n+1}_{e^*_{n+1}})\\ &\tilde{\leq}(\alpha,\phi)(f_e,h^n_{e^*_n})\tilde{d}((T,\varphi)f_e,(T,\varphi)h^n_{e^*_n})\\ &\tilde{\leq}\psi(\tilde{d}(f_e,h^n_{e^*_n})).\end{aligned}$$

This implies that

$$\tilde{d}(f_e,h^{n+1}_{e^*_{n+1}})\tilde{\leq}\psi(\tilde{d}(f_e,h^n_{e^*_n})), \text{ for all } n\in\mathbb{N}.$$

Thus

$$\tilde{d}(f_e,h^n_{e^*_n})\tilde{\leq}\psi^n(\tilde{d}(f_e,h^n_{e^*_n})), \text{ for all } n\geq 1.$$

So, letting $n\to\infty$, we have

$$(3.7)\qquad h^n_{e^*_n}\to f_e.$$

Similarly, using the two inequalities (3.6) and (3.1), we get

$$(3.8)\qquad h^n_{e^*_n}\to g_{\acute{e}}$$

Using the two inequalities (3.7) and (3.8), the uniqueness of limit gives us $f_e = g_{\acute{e}}$. Hence we proved that f_e is the unique fixed point of (T,φ). □

Definition 3.9. Let $FSC(\tilde{E})$ be collection of all fuzzy soft points of $\tilde{E}$ and let $T: FSC(\tilde{E})\to FSC(\tilde{E})$ and $\alpha,\beta: FSC(\tilde{E})\to\mathcal{R}(E)^*$ be two mappings. We say that T is a cyclic fuzzy soft (α,β)-admissible mapping, if

$$f_e\tilde{\in}FSC(\tilde{E}), \alpha(f_e)\tilde{\leq}\tilde{1}\Rightarrow\beta(Tf_e)\tilde{\leq}\tilde{1}$$

and

$$f_e\tilde{\in}FSC(\tilde{E}), \beta(f_e)\tilde{\leq}\tilde{1}\Rightarrow\alpha(Tf_e)\tilde{\leq}\tilde{1}.$$

Definition 3.10. Let $(\tilde{E},\tilde{d})$ be a fuzzy soft metric space and $(T,\varphi):(\tilde{E},\tilde{d})\to(\tilde{E},\tilde{d})$ be a given fuzzy soft mapping. Then we say that (T,φ) is fuzzy soft (α,β)-Banach contractive mapping, if there exists two fuzzy soft functions $(\alpha,\psi),(\beta,\alpha): FSC(\tilde{E})\to\mathcal{R}(E)^*$ and $\tilde{0}\leq\tilde{\tilde{r}}<\tilde{1}$ such that

$$(3.9)\qquad (\alpha,\psi)(f_e)(\beta,\phi)(g_{\acute{e}})((T,\varphi)f_e,(T,\varphi)g_{\acute{e}})\tilde{\leq}\tilde{\tilde{r}}\cdot\tilde{d}(f_e,g_{\acute{e}}), \qquad \forall f_e,g_{\acute{e}}\tilde{\in}FSC(\tilde{E}).$$

Next, we give some fixed point result for fuzzy soft (α,β)-Banach-contraction mappings in complete fuzzy soft metric space.

Theorem 3.11. *Let $(\tilde{E},\tilde{d})$ be a complete fuzzy soft metric space . Let (T,φ) be a fuzzy soft (α,β)-Banach contractive mapping from $FSC(\tilde{E})$ into itself satisfying the following:*

(i) *there exists $f^0_{e_0}\tilde{\in}FSC(\tilde{E})$ such that $(\alpha,\psi)(f^0_{e_0})\tilde{\leq}\tilde{1}$ and $(\beta,\phi)(f^0_{e_0})\tilde{\leq}\tilde{1}$,*
(ii) *(T,φ) is a cyclic fuzzy soft (α,β)-admissible,*

(iii) *one of the following conditions holds:*
(3_a) (T,φ) is fuzzy soft continuous,
(3_b) if $\{f^n_{e_n}\}$ is a sequence in $(\tilde{E},\tilde{d})$ such that $\{f^n_{e_n}\}\to f_e\tilde{\in}FSC(\tilde{E})$ as $n\to\infty$ and $(\beta,\phi)(f^n_{e_n})\tilde{\preceq}\tilde{1}$ for all $n\in\mathbb{N}$, then $(\beta,\phi)(f_e)\tilde{\preceq}\tilde{1}$.

Then (T,φ) has a fixed point. Furthermore, if $(\alpha,\psi)(f_e)\tilde{\preceq}\tilde{1}$ and $(\beta,\phi)(f_e)\tilde{\preceq}\tilde{1}$ for all fixed point $f_e\tilde{\in}FSC(\tilde{E})$, then (T,φ) has a unique fixed point.

Proof. Let$f^0_{e_0}\tilde{\in}FSC(\tilde{E})$ such that $(\alpha,\psi)(f^0_{e_0})\tilde{\preceq}\tilde{1}$ and $(\beta,\phi)(f^0_{e_0})\tilde{\preceq}\tilde{1}$. We will construct the iterative sequence $\{f^n_{e_n}\}$, where $f^n_{e_n}=(T,\varphi)f^{n-1}_{e_{n-1}}$ for all $n\in\mathbb{N}$. Since (T,φ) is a cyclic fuzzy soft (α,β)-admissible mapping, we have

$$(\alpha,\psi)f^0_{e_0}\tilde{\preceq}\tilde{1}\Rightarrow(\beta,\phi)f^1_{e_1}=(\beta,\phi)((T,\varphi)f^0_{e_0})\tilde{\preceq}\tilde{1}\tag{3.10}$$

and

$$(\beta,\phi)f^0_{e_0}\tilde{\preceq}\tilde{1}\Rightarrow(\alpha,\psi)f^1_{e_1}=(\alpha,\psi)((T,\varphi)f^0_{e_0})\tilde{\preceq}\tilde{1}.\tag{3.11}$$

By similar method, we get

$$(\alpha,\psi)f^n_{e_n}\tilde{\preceq}\tilde{1}\text{ and }(\beta,\phi)f^n_{e_n}\tilde{\preceq}\tilde{1}\text{ for all }n\in\mathbb{N}$$

This implies that

$$(\alpha,\psi)f^{n-1}_{e_{n-1}}(\beta,\phi)f^n_{e_n}\tilde{\preceq}\tilde{1}\text{ for all }n\in\mathbb{N}.$$

From the fuzzy soft (α,β)-Banach contractive condition of (T,φ), we have

$$\begin{aligned}\tilde{d}(f^n_{e_n},f^{n+1}_{e_{n+1}})&=\tilde{d}((T,\varphi)f^{n-1}_{e_{n-1}},(T,\varphi)f^n_{e_n})\\&\tilde{\preceq}(\alpha,\psi)f^{n-1}_{e_{n-1}}(\beta,\phi)f^n_{e_n}\cdot\tilde{d}((T,\varphi)f^{n-1}_{e_{n-1}},(T,\varphi)f^n_{e_n})\\&\tilde{\preceq}\tilde{\tilde{r}}\cdot\tilde{d}(f^{n-1}_{e_{n-1}},f^n_{e_n})\\&\tilde{\preceq}...\tilde{\preceq}\tilde{\tilde{r}}^n\cdot\tilde{d}(f^0_{e_0},f^1_{e_1}),\text{ for all }n\in\mathbb{N}.\end{aligned}$$

Let $n,m\in\mathbb{N}$ such that $n>m$. Then we get

$$\begin{aligned}\tilde{d}(f^m_{e_m},f^n_{e_n})&\tilde{\preceq}\tilde{d}(f^m_{e_m},f^{m+1}_{e_{m+1}})+\tilde{d}(f^{m+1}_{e_{m+1}},f^{m+2}_{e_{m+2}})+...+\tilde{d}(f^{n-1}_{e_{n-1}},f^n_{e_n})\\&\tilde{\preceq}(\tilde{\tilde{r}}^m+\tilde{\tilde{r}}^{m+1}+...+\tilde{\tilde{r}}^{n-1})\cdot\tilde{d}(f^0_{e_0},f^1_{e_1})\\&\tilde{\preceq}\frac{\tilde{\tilde{r}}^m}{\tilde{1}-\tilde{\tilde{r}}}\cdot\tilde{d}(f^0_{e_0},f^1_{e_1}).\end{aligned}$$

Thus this implies
$\tilde{d}(f^m_{e_m},f^n_{e_n})\to\tilde{0}$ as $(m,n\to\infty)$.
So $\{f^m_{e_m}\}$ is a fuzzy soft Cauchy sequence, by the completeness of $(\tilde{E},\tilde{d})$, there is a fuzzy soft point $f_e\tilde{\in}FSC(\tilde{E})$ such that $f^n_{e_n}\to f_e$ as $(n\to\infty)$.

Now we assume that (T,φ) is fuzzy soft continuous. Then, we obtain

$$f_e=lim_{n\to\infty}f^{n+1}_{e_{n+1}}=lim_{n\to\infty}(T,\varphi)f^n_{e_n}=(T,\varphi)(lim_{n\to\infty}f^n_{e_n})=(T,\varphi)f_e.$$

Now we will assume that the condition (3_b) holds. Then $(\beta,\phi)f_e\tilde{\leq}\tilde{1}$. Thus we have, for each $n\in\mathbb{N}$,

$$\begin{aligned}\tilde{d}((T,\varphi)f_e,f_e)&\tilde{\leq}\tilde{d}((T,\varphi)f_e,(T,\varphi)f^n_{e_n})+\tilde{d}((T,\varphi)f^n_{e_n},f_e)\\&\tilde{\leq}(\alpha,\psi)f^n_{e_n}(\beta,\phi)f_e\cdot\tilde{d}((T,\varphi)f_e,(T,\varphi)f^n_{e_n})+\tilde{d}((T,\varphi)f^n_{e_n},f_e)\\&\tilde{\leq}\tilde{\tilde{r}}\cdot\tilde{d}(f^n_{e_n},f_e)+\tilde{d}(f^{n+1}_{e_{n+1}},f_e).\end{aligned}$$

Letting $n\to\infty$, we get $\tilde{d}((T,\varphi)f_e,f_e)=\tilde{0}$, that is $(T,\varphi)f_e=f_e$. This shows that f_e is a fixed point of (T,φ).

Now, we show that f_e is the unique fixed point of (T,φ). Assume that $g_{\acute{e}}$ is another fixed point of (T,φ). From hypothesis, we find that $(\alpha,\psi)f_e\tilde{\leq}\tilde{1}$ and $(\beta,\phi)g_{\acute{e}}\tilde{\leq}\tilde{1}$, and hence

$$\begin{aligned}\tilde{d}(f_e,g_{\acute{e}})&=\tilde{d}((T,\varphi)f_e,(T,\varphi)g_{\acute{e}})\\&\tilde{\leq}(\alpha,\psi)f_e(\beta,\phi)g_{\acute{e}}\cdot\tilde{d}((T,\varphi)f_e,(T,\varphi)g_{\acute{e}})\\&\tilde{\leq}\tilde{\tilde{r}}\cdot\tilde{d}(f_e,g_{\acute{e}}).\end{aligned}$$

This shows that $\tilde{d}(f_e,g_{\acute{e}})=\tilde{0}$ and then $f_e=g_{\acute{e}}$. Therefore, f_e is the unique fixed point of (T,φ). □

Now generalized cyclic fuzzy soft (α,β)-contractive type mappings as follows

Definition 3.12. Let $(\tilde{E},\tilde{d})$ be a fuzzy soft metric space and $(T,\varphi):(\tilde{E},\tilde{d})\to(\tilde{E},\tilde{d})$ be a given fuzzy soft mapping. Then we say that (T,φ) is generalized fuzzy soft (α,β)-contraction, if there exists two fuzzy soft functions $(\alpha,\psi),(\beta,\phi):FSC(\tilde{E})\to\mathcal{R}(E)^*$ and $\tilde{0}\leq\tilde{\tilde{r}}<\tilde{1}$ such that

$$(\alpha,\psi)f_e(\beta,\phi)g_{\acute{e}}\cdot\tilde{d}((T,\varphi)f_e,(T,\varphi)g_{\acute{e}})\tilde{\leq}\tilde{\tilde{r}}\cdot\hat{d}(f_e,g_{\acute{e}}),\tag{3.12}$$

where

$$\begin{aligned}\hat{d}(f_e,g_{\acute{e}})=&\max\{\tilde{d}(f_e,g_{\acute{e}}),\tilde{d}((T,\varphi)f_e,f_e),\tilde{d}((T,\varphi)g_{\acute{e}},g_{\acute{e}}),\\&\tfrac{1}{2}[\tilde{d}((T,\varphi)f_e,g_{\acute{e}})+\tilde{d}(f_e,(T,\varphi)g_{\acute{e}})]\},\end{aligned}$$

for all $f_e,g_{\acute{e}}\tilde{\in}FSC(\tilde{E})$.

Theorem 3.13. *Let $(\tilde{E},\tilde{d})$ be a complete fuzzy soft metric space . Let (T,φ) be a generalized fuzzy soft (α,β)-Banach contractive mapping from $FSC(\tilde{E})$ into itself satisfying the following:*

(i) *there exists $f^0_{e_0}\tilde{\in}FSC(\tilde{E})$ such that $(\alpha,\psi)(f^0_{e_0})\tilde{\leq}\tilde{1}$ and $(\beta,\phi)(f^0_{e_0})\tilde{\leq}\tilde{1}$,*
(ii) *(T,φ) is a cyclic fuzzy soft (α,β)-admissible;*
(iii) *one of the following conditions holds:*
(3_a) (T,φ) is fuzzy soft continuous,
(3_b) if $\{f^n_{e_n}\}$ is a sequence in $(\tilde{E},\tilde{d})$ such that $\{f^n_{e_n}\}\to f_e\tilde{\in}FSC(\tilde{E})$ as $n\to\infty$ and $(\beta,\phi)(f^n_{e_n})\tilde{\leq}\tilde{1}$, for all $n\in\mathbb{N}$, then $(\beta,\phi)(f_e)\tilde{\leq}\tilde{1}$.

Then (T,φ) has a fixed point. Furthermore, if $(\alpha,\psi)(f_e)\tilde{\lesssim}\tilde{1}$ and $(\beta,\phi)(f_e)\tilde{\lesssim}\tilde{1}$, for all fixed point $f_e\tilde{\in}FSC(\tilde{E})$, then (T,φ) has a unique fixed point.

Proof. Let$f^0_{e_0}\tilde{\in}FSC(\tilde{E})$ such that $(\alpha,\psi)(f^0_{e_0})\tilde{\lesssim}\tilde{1}$ and $(\beta,\phi)(f^0_{e_0})\tilde{\lesssim}\tilde{1}$. We will construct the iterative sequence $\{f^n_{e_n}\}$, where $f^n_{e_n}=(T,\varphi)f^{n-1}_{e_{n-1}}$, for all $n\in\mathbb{N}$. Since (T,φ) is a cyclic fuzzy soft (α,β)-admissible mapping, we have

$$(\alpha,\psi)f^0_{e_0}\tilde{\lesssim}\tilde{1}\Rightarrow(\beta,\phi)f^1_{e_1}=(\beta,\phi)((T,\varphi)f^0_{e_0})\tilde{\lesssim}\tilde{1} \tag{3.13}$$

and

$$(\beta,\phi)f^0_{e_0}\tilde{\lesssim}\tilde{1}\Rightarrow(\alpha,\psi)f^1_{e_1}=(\alpha,\psi)((T,\varphi)f^0_{e_0})\tilde{\lesssim}\tilde{1}. \tag{3.14}$$

by similar method, we get

$$(\alpha,\psi)f^n_{e_n}\tilde{\lesssim}\tilde{1} \text{ and } (\beta,\phi)f^n_{e_n}\tilde{\lesssim}\tilde{1}, \text{ for all } n\in\mathbb{N}$$

This implies that

$$(\alpha,\psi)f^{n-1}_{e_{n-1}}(\beta,\phi)f^n_{e_n}\tilde{\lesssim}\tilde{1}, \text{ for all } n\in\mathbb{N}.$$

From the generalized fuzzy soft (α,β)-contractive condition of (T,φ), we have

$$\begin{aligned}\tilde{d}(f^n_{e_n},f^{n+1}_{e_{n+1}})&=\tilde{d}((T,\varphi)f^{n-1}_{e_{n-1}},(T,\varphi)f^n_{e_n})\\&\tilde{\lesssim}(\alpha,\psi)f^{n-1}_{e_{n-1}}(\beta,\phi)f^n_{e_n}\cdot\tilde{d}((T,\varphi)f^{n-1}_{e_{n-1}},(T,\varphi)f^n_{e_n})\\&\tilde{\lesssim}\tilde{\tilde{r}}\cdot\hat{d}(f^{n-1}_{e_{n-1}},f^n_{e_n}),\text{ for all } n\in\mathbb{N},\end{aligned}$$

where

$$\begin{aligned}\hat{d}(f^{n-1}_{e_{n-1}},f^n_{e_n})=&\max\{\tilde{d}(f^n_{e_n},f^{n-1}_{e_{n-1}}),\tilde{d}((T,\varphi)f^{n-1}_{e_{n-1}},f^{n-1}_{e_{n-1}}),\tilde{d}((T,\varphi)f^n_{e_n},f^n_{e_n}),\\&\tfrac{1}{2}[\tilde{d}((T,\varphi)f^{n-1}_{e_{n-1}},f^n_{e_n})+\tilde{d}(f^{n-1}_{e_{n-1}},(T,\varphi)f^n_{e_n})]\}\\=&\max\{\tilde{d}(f^n_{e_n},f^{n-1}_{e_{n-1}}),\tilde{d}(f^n_{e_n},f^{n-1}_{e_{n-1}}),\tilde{d}(f^{n+1}_{e_{n+1}},f^n_{e_n}),\\&\tfrac{1}{2}[\tilde{d}(f^n_{e_n},f^n_{e_n})+\tilde{d}(f^{n-1}_{e_{n-1}},f^{n+1}_{e_{n+1}})]\}\\\tilde{\lesssim}&\max\{\tilde{d}(f^n_{e_n},f^{n-1}_{e_{n-1}}),\tilde{d}(f^n_{e_n},f^{n+1}_{e_{n+1}})\}.\end{aligned}$$

Thus $\tilde{d}(f^n_{e_n},f^{n+1}_{e_{n+1}})\tilde{\lesssim}\tilde{\tilde{r}}\cdot$ max $\{\tilde{d}(f^n_{e_n},f^{n-1}_{e_{n-1}}),\tilde{d}(f^n_{e_n},f^{n+1}_{e_{n+1}})\}$.

Suppose $\tilde{d}(f^n_{e_n},f^{n+1}_{e_{n+1}})$ is maximum. Then

$$\begin{aligned}\tilde{d}(f^n_{e_n},f^{n+1}_{e_{n+1}})&\tilde{\lesssim}\tilde{\tilde{r}}\cdot\tilde{d}(f^n_{e_n},f^{n+1}_{e_{n+1}})\\&\tilde{\lesssim}\tilde{d}(f^n_{e_n},f^{n+1}_{e_{n+1}}) \text{ is a contradiction.}\end{aligned}$$

Thus $\tilde{d}(f^n_{e_n},f^{n+1}_{e_{n+1}})\tilde{\lesssim}\tilde{\tilde{r}}\cdot\tilde{d}(f^n_{e_n},f^{n-1}_{e_{n-1}})$, for all $n\in\mathbb{N}$.

Let $n,m\in\mathbb{N}$ such that $n>m$. Then we get

$$\tilde{d}(f^m_{e_m},f^n_{e_n})\tilde{\lesssim}\tilde{d}(f^m_{e_m},f^{m+1}_{e_{m+1}})+\tilde{d}(f^{m+1}_{e_{m+1}},f^{m+2}_{e_{m+2}})+...+\tilde{d}(f^{n-1}_{e_{n-1}},f^n_{e_n})$$

$$\tilde{\leq}(\tilde{\tilde{r}}^{m}+\tilde{\tilde{r}}^{m+1}+...+\tilde{\tilde{r}}^{n-1})\cdot\tilde{d}(f^{0}_{e_0},f^{1}_{e_1})$$

$$\tilde{\leq}\frac{\tilde{\tilde{r}}^{m}}{\tilde{1}-\tilde{\tilde{r}}}\cdot\tilde{d}(f^{0}_{e_0},f^{1}_{e_1}).$$

Thus this implies

$$\tilde{d}(f^{m}_{e_m},f^{n}_{e_n})\to\tilde{0} \text{ as } (m,n\to\infty).$$

So $\{f^{m}_{e_m}\}$ is a fuzzy soft Cauchy sequence, by the completeness of $(\tilde{E},\tilde{d})$, there is a fuzzy soft point $f_e\tilde{\in}FSC(\tilde{E})$ such that $f^{n}_{e_n}\to f_e$ as $(n\to\infty)$.

Now we assume that (T,φ) is fuzzy soft continuous. Then, we obtain

$$f_e=lim_{n\to\infty}f^{n+1}_{e_{n+1}}=lim_{n\to\infty}(T,\varphi)f^{n}_{e_n}=(T,\varphi)(lim_{n\to\infty}f^{n}_{e_n})=(T,\varphi)f_e.$$

Now we will assume that the condition (3_b) holds. Then $(\beta,\phi)f_e\tilde{\leq}\tilde{1}$. Thus we have, for each $n\in\mathbb{N}$,

$$\tilde{d}((T,\varphi)f_e,f_e)\tilde{\leq}\tilde{d}((T,\varphi)f_e,(T,\varphi)f^{n}_{e_n})+\tilde{d}((T,\varphi)f^{n}_{e_n},f_e)$$

$$\tilde{\leq}(\alpha,\psi)f^{n}_{e_n}(\beta,\phi)f_e\cdot\tilde{d}((T,\varphi)f_e,(T,\varphi)f^{n}_{e_n})+\tilde{d}((T,\varphi)f^{n}_{e_n},f_e)$$

$$\tilde{\leq}\tilde{\tilde{r}}\cdot\,\hat{d}(f^{n}_{e_n},f_e)+\tilde{d}(f^{n+1}_{e_{n+1}},f_e)$$

$$=\tilde{\tilde{r}}\cdot\,\max\{\tilde{d}(f^{n}_{e_n},f_e),\tilde{d}((T,\varphi)f^{n}_{e_n},f^{n}_{e_n}),\tilde{d}((T,\varphi)f_e,f_e),$$

$$\frac{1}{2}[\tilde{d}((T,\varphi)f^{n}_{e_n},f_e),\tilde{d}(f^{n}_{e_n},(T,\varphi)f_e)]\}+\tilde{d}(f^{n+1}_{e_{n+1}},f_e).$$

Letting $n\to\infty$, we get

$$\tilde{d}((T,\varphi)f_e,f_e)\tilde{\leq}\tilde{\tilde{r}}\cdot\tilde{d}((T,\varphi)f_e,f_e)$$

$$\tilde{\leq}\tilde{d}((T,\varphi)f_e,f_e).$$

This is a contradiction. Then $(T,\varphi)f_e=f_e$. This shows that f_e is a fixed point of (T,φ).

Now, we show that f_e is the unique fixed point of (T,φ). Assume that $g_{\acute{e}}$ is another fixed point of (T,φ). From hypothesis, we find that $(\alpha,\psi)f_e\tilde{\geq}\tilde{0}$ and $(\beta,\phi)g_{\acute{e}}\tilde{\geq}\tilde{0}$. Then

$$\tilde{d}(f_e,g\acute{e})=\tilde{d}((T,\varphi)f_e,(T,\varphi)g_{\acute{e}})$$

$$\tilde{\leq}(\alpha,\psi)f_e(\beta,\phi)g\acute{e}\cdot\tilde{d}((T,\varphi)f_e,(T,\varphi)g_{\acute{e}})$$

$$\tilde{\leq}\tilde{\tilde{r}}\cdot\hat{d}(f_e,g_{\acute{e}})$$

$$=\tilde{\tilde{r}}\cdot\max\{\tilde{d}(f_e,g\acute{e}),\tilde{d}((T,\varphi)f_e,f_e),\tilde{d}((T,\varphi)g_{\acute{e}},g_{\acute{e}}),$$

$$\frac{1}{2}[\tilde{d}((T,\varphi)f_e,g_{\acute{e}})+\tilde{d}(f_e,(T,\varphi)g_{\acute{e}})]\}$$

$$=\tilde{\tilde{r}}\cdot\max\{\tilde{d}(f_e,g_{\acute{e}}),\tilde{d}(f_e,f_e),\tilde{d}(g_{\acute{e}},g_{\acute{e}}),\frac{1}{2}[\tilde{d}(f_e,g_{\acute{e}})+\tilde{d}(f_e,g_{\acute{e}})]\}$$

$$\tilde{\leq}\tilde{\tilde{r}} \cdot \tilde{d}(f_e, g_{\acute{e}})$$

$$\tilde{<}\tilde{d}(f_e, g_{\acute{e}}).$$

This is a contradiction. Thus $f_e = g_{\acute{e}}$. So, f_e is the unique fixed point of (T, φ). □

4. Conclusions

In this paper, we present the notions of fuzzy soft $\alpha - \psi$-contractive mappings and cyclic fuzzy soft $(\alpha, \beta) - \psi$-contractive mappings in fuzzy soft metric spaces and some fixed points theorems has been proved. In the future, we will try to improve the search performance further for some new types of mappings.

Acknowledgements. The authors are very grateful to the Editors-in-Chief and the Managing Editors for their valuable advice. The authors are also expressed their sincere thanks to the the anonymous Referees for their valuable suggestions in rewriting this paper in the present form.

References

[1] M. Abbas, M. Jun and D. Gopal, ψ-weak contractions in fuzzy metric spaces, Iranian Journal of Fuzzy Systems 6 (2011) 141–148.
[2] B. Ahmad and Athar Kharal, On Fuzzy Soft Sets, Advances in Fuzzy Systems, Volume 2009, Article ID 586507, 6 pages.
[3] M. I. Ali, F. Feng, X. Liu, W. K. Min and M. Shabir, On some new operations in soft set theory, Comput. Math. Appl. 57 (2009) 1547–1553.
[4] Ritu Arora and Mohit Kumar, Unique fixed point theorems for α-ψ-contractive type mappings in fuzzy metric space, Cogent Mathematics 3 (1) (2016) pages 1183286. http://dx.doi.org/10.1080/23311835.2016.1183286
[5] Thangaraj Beaulaa and Christinal Gunaseeli, On fuzzy soft metric spaces, Malaya J. Mat. 2 (3) (2014) 197–202.
[6] Thangaraj Beaulaa and R.Raja, Completeness in Fuzzy Soft Metric Space, Malaya J. Mat. S (2) (2015) 438–442.
[7] N. Çaĝman and S. Enginoĝlu, Soft set theory and uni-int decision making, Eur. J. Oper. Res. 207 (2010) 848–855.
[8] N. Çaĝman and S. Enginoğlu, Soft matrix theory and its decision making, Comput. Math. Appl. 59 (2010), 3308–3314.
[9] N. Çağman, F. Çitak and S. Enginoğlu, Fuzzy parameterized fuzzy soft set theory and its applications, Turk. J. Fuzzy Syst. 1 (2010) 21–35.
[10] N. Çağman, F. Çitak, S. Enginoğlu, FP-soft set theory and its applications, Ann. Fuzzy Math. Inform. 2 (2) (2011) 219–226.
[11] S. Christinal Gunaseeli, Some Contributions to Special Fuzzy Topological Spaces, chapter 5, Thesis submitted to the Bharathidasan University, India, Tiruchirappalli in partial fulfillment of the requirements for the Degree of Doctor Of Philosophy in Mathematics, Ref.No.16690/Ph.D. /Mathematics/Full-Time/July (2012).
[12] S. Das and S.K. Samanta, Soft real sets, soft real numbers and their properties, J. Fuzzy Math. 20 (3) (2012) 551–576.
[13] F. Feng, Y. B. Jun, X. Liu and L. Li, An adjustable approach to fuzzy soft set based decision making, J. Comput. App. Math. 234 (2010) 10–20.
[14] F. Feng, X. Y. Liu, V. Leoreanu-Fotea and Y. B. Jun, Soft sets and soft rough sets, Inform. Sci. 181 (2011) 1125–1137.
[15] D. Gopal and C. Vetro, Some new fixed point theorems in fuzzy metric spaces, Iranian Journal of Fuzzy Systems 11 (2014) 95–107.

[16] D. Gopal, M. Imded, C. Vetro and M. Hasan, Fixed point theory for cyclic weak ϕ-contraction in fuzzy metric spaces, Journal of Nonlinear Analysis and Application (2012) 1–11, Article ID: jnaa-00110.

[17] M. Grabiec, Fixed points in fuzzy metric spaces, Fuzzy Sets and Systems 27 (1988) 385–389.

[18] V. Gregori and A. Sapena, On fixed point theorems in fuzzy metric spaces, Fuzzy Sets and Systems 125 (2002) 245–252.

[19] S. Hong, Fixed points for modified fuzzy ψ-contractive set-valued mappings in fuzzy metric spaces, Fixed Point Theory and Applications 1 (2014) 1–12.

[20] J. Mahanta and P. K. Das, Results on fuzzy soft topological spaces, (2012), arXiv:1203.0634v1.

[21] P. K. Maji, R. Biswas and A. R. Roy, Fuzzy soft sets, J. Fuzzy Math. 9 (3) (2001) 589–602.

[22] P. K. Maji, A. R. Roy and R. Biswas, An application of soft sets in a decision making problem, Comput. Math. Appl. 44 (8-9) (2002) 1077–1083.

[23] P. K. Maji, R. Biswas and A. R. Roy, Soft set theory, Computers and Mathematics with Applications 45 (2003) 555–562.

[24] P. Majumdar and S. K. Samanta, Generalised fuzzy soft sets, Comput. Math. Appl. 59 (2010) 1425–1432.

[25] D. Miheţ, A Banach contraction theorem in fuzzy metric spaces, Fuzzy Sets and Systems 144 (2004) 431–439.

[26] D. Miheţ, On fuzzy contractive mappings in fuzzy metric spaces. Fuzzy Sets and Systems 158 (2007) 915–921.

[27] D. Miheţ, Fuzzy ψ-contractive mappings in non-Archimedean fuzzy metric spaces, Fuzzy Sets and Systems 159 (2008) 739–744.

[28] D. Molodtsov, Soft set theory First results, Comput. Math. Appl. 8 (4/5) (1999) 19–31.

[29] M. Mursaleen, S. A. Mohiuddine and R. P. Aggarwal, Coupled fixed point theorems for $\alpha-\psi$-contractive type mappings in partially order metric spaces, Fixed Point Theory and Application, Fixed Point Theory and Applications (2012), 2012:228. http://www.fixedpointtheoryandapplications.com/content/2012/1/228

[30] M. Mursaleen, H. M. Srivastava and S. K. Sharma, Generalized statistically convergent sequences of fuzzy numbers, Journal of Intelligent and Fuzzy Systems 30 (2016) 1511–1518.

[31] S. Phiangsungnoen, W. Sintunavarat and P. Kumam, Fuzzy fixed point theorems in Hausdorff fuzzy metric spaces, Journal of Inequalities and Applications 1 (2014) 1–10.

[32] A. R. Roy and P. K. Maji, A fuzzy soft set theoretic approach to decision making problems, J. Comput. Appl. Math. 203 (2007) 412–418.

[33] S. Roy and T. K. Samanta, A note on fuzzy soft topological spaces, Annals of Fuzzy Mathematics and informatics 3 (2) (2012) 305–311.

[34] B. Samet, C. Vetro and P. Vetro, Fixed point theorems for $\alpha-\psi$-contractive type mappings, Nonlinear Analysis: Theory, Methods and Applications 75 (4) (2012) 2154–2165.

[35] A. F. Sayed and Jamshaid Ahmad, Some Fixed Point Theorems For Fuzzy Soft Contractive Mappings In Fuzzy Soft Metric Spaces, an article accepted at Italian Journal of Pure and Applied Mathematics will appear in n.39 (2018).

[36] A. Sezgin and A. O. Atagün, On operations of soft sets, Comput. Math. Appl. 61 (2011) 1457–1467.

[37] B. Tanay and M. B. Kandemir, Topological structures of fuzzy soft sets, Comput. Math. Appl. 61 (2011) 412–418.

[38] Z. Xiao, K. Gong and Y. Zou, Acombined forecasting approach based on fuzzy soft sets, J. Comput. Appl. Math. 228 (2009) 326–333.

[39] J. Xu, , H. He and H. Man, DCPE co-training for classification, Neuro Computing 86 (2012) 75–85.

[40] J. Zhan and Y. B. Jun, Soft BL-algebras based on fuzzy sets, Comput. Math. Appl. 59 (2010) 2037–2046.

[41] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338–353.

[42] Y. Zou and Z. Xiao, Data analysis approaches of soft sets under incomplete information, Knowledge-Based Systems 21 (2008) 941–945.

A. F. Sayed (dr.afsayed@hotmail.com, afssayed@uqu.edu.sa)
Mathematics Department, Al-Lith University College, Umm Al-Qura University, P.O. Box 112, Al-Lith 21961, Makkah Al Mukarramah, Kingdom of Saudi Arabia

A. Alahmari (aaahmari@uqu.edu.sa)
Mathematics Department, College of Applied Sciences, Umm Al-Qura University, Kingdom of Saudi Arabia